

The cooling of CO white dwarfs: influence of the internal chemical distribution

Maurizio Salaris^{1,2}, Inmaculada Domínguez³, Enrique García-Berro⁴, Margarida Hernanz¹,
Jordi Isern¹, Robert Mochkovitch⁵

Received _____; accepted _____

¹Institut d'Estudis Espacials de Catalunya, CSIC Research Unit, Edifici Nexus-104, C/
Gran Capità 2-4, 08034 Barcelona, Spain

²Max-Planck-Institut für Astrophysik, Karl Schwarzschild Strasse 1, 85740 Garching,
Germany

³Departamento de Física Teórica y del Cosmos, Universidad de Granada, Facultad de
Ciencias, 18071 Granada, Spain

⁴Departament de Física Aplicada, Universitat Politècnica de Catalunya, Jordi Girona
Salgado s/n, Mòdul B-5, Campus Nord, 08034 Barcelona, Spain

⁵Institut d'Astrophysique de Paris, C.N.R.S., 98 bis Bd. Arago, 75014 Paris, France

ABSTRACT

White dwarfs are the remnants of stars of low and intermediate masses on the main sequence. Since they have exhausted all their nuclear fuel, their evolution is just a gravothermal process. The release of energy only depends on the detailed internal structure and chemical composition and on the properties of the envelope equation of state and opacity; its consequences on the cooling curve (i.e. the luminosity versus time relationship) depend on the luminosity at which this energy is released.

The internal chemical profile depends on the rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction as well as on the treatment of convection. High reaction rates produce white dwarfs with oxygen rich cores surrounded by carbon rich mantles. This reduces the available gravothermal energy and decreases the lifetime of white dwarfs.

In this paper we compute detailed evolutionary models providing chemical profiles for white dwarfs having progenitors in the mass range from 1.0 to $7 M_{\odot}$ and we examine the influence of such profiles in the cooling process. The influence of the process of separation of carbon and oxygen during crystallization is decreased as a consequence of the initial stratification, but it is still important and cannot be neglected. As an example, the best fit to the luminosity functions of Liebert et al. (1988) and Oswalt et al. (1996) gives an age of the disk of 9.3 and 11.0 Gyr, respectively, when this effect is taken into account, and only 8.3 and 10.0 Gyrs when it is neglected.

Subject headings: stars: interiors — stars: white dwarfs

1. Introduction

The final result of the evolution of low and intermediate mass stars ($M \lesssim 7\text{--}8M_{\odot}$) is a carbon-oxygen white dwarf. Since these stars have exhausted all their sources of nuclear energy, their evolution is determined by the gravothermal adjustment of their interiors induced by energy losses. As shown by Koester and Chanmugam (1990) using the virial theorem, this evolution can be interpreted in terms of a cooling process. The rate of cooling is determined, among other factors, by the ionic specific heat which depends on the relative proportions of carbon and oxygen. The change of chemical composition between the solid and the liquid at the onset of crystallization and the gravitationally induced redistribution of carbon and oxygen provides an additional source of energy (Mochkovitch 1983, García-Berro et al. 1988), the importance of which depends, among other things, on the shape of the phase diagram.

Segretain et al. (1994) computed detailed cooling sequences using the most up to date input physics (both for the equation of state and the phase diagram), and taking into account both the release of latent heat and the release of gravitational energy induced by the redistribution of carbon and oxygen upon crystallization. The main result of including this extra energy source was a noticeable increase in the cooling times. For instance, the time taken by a typical $0.6 M_{\odot}$ white dwarf with equal mass fractions of carbon and oxygen to reach a luminosity $\log(L/L_{\odot}) = -4.5$ was 11.5 Gyr, when the redistribution process was properly taken into account, instead of 9.2 Gyr when this effect was neglected (a correction of $\sim 20\%$).

Two aspects that can reduce the efficiency of chemical redistribution arise from the assumptions that the liquid mantle surrounding the solid core is always perfectly mixed and that the white dwarf is initially made of half carbon and half oxygen uniformly distributed throughout the star. Although the validity of the first point was early studied

by Mochkovitch (1983), it will be the object of an updated analysis in a forthcoming paper (Isern et al. 1996), and here we will concentrate on the role played by different chemical initial profiles.

Mazzitelli & D’Antona (1986a,b; 1987) studied the evolution from the main sequence to the white dwarf stage for 1, 3 and 5 M_{\odot} stars and found that the composition (and in particular the carbon-oxygen ratio) of the resulting white dwarf was very sensitive to the adopted cross section for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, to the detailed prescriptions adopted for convective mixing, and to the mass of the star on the main sequence. Their results indicate that a typical 0.6 M_{\odot} white dwarf consists of an inner oxygen-rich core surrounded by a carbon-rich mantle, whereas a 1.0 M_{\odot} white dwarf has almost flat carbon-oxygen profiles with roughly $X_{\text{C}}=X_{\text{O}}=0.5$ (by mass). This has two effects: first, since oxygen crystallizes at higher temperatures than carbon, both the latent heat and the gravitational energy are released at higher luminosities and the induced delay in the cooling times is smaller. Second, since the inner core is oxygen-rich the gravitational energy released upon crystallization is smaller, thus reducing the effect of the redistribution process. The profiles obtained by Mazzitelli & D’Antona (1986a,b) were also adopted by Segretain et al. (1994), who found that the time taken by a 0.6 M_{\odot} white dwarf to reach $\log(L/L_{\odot}) = -4.5$ was 8.8 Gyr or 10.0 Gyr, depending on whether the redistribution process was neglected or taken into account. Thus, the delay induced by phase separation was 1.2 Gyr, which is a correction of $\sim 15\%$.

The purpose of this paper is to clarify, in light of the new determinations of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate, whether or not the interiors of white dwarfs are stratified before crystallization sets in and to determine the effect of the actual chemical profile on the cooling times, thereby providing better estimates of the ages of white dwarfs and of the age of the solar neighborhood.

2. Input physics

The evolutionary stellar models presented in this paper have been computed using the evolutionary code FRANEC (Frascati RAPHSON Newton Evolutionary Code), as described in Chieffi & Straniero (1989); the reader is referred to this paper for an exhaustive discussion about the physical inputs adopted in the code. In the following we will discuss briefly only some basic features relevant for the scope of this paper.

The boundaries of convective regions are set by adopting the Schwarzschild criterion and no mechanical overshooting is allowed. Semiconvection during central helium burning is computed according to the method described in Castellani et al. (1985), and the breathing pulses occurring during the last portion of core helium burning have been inhibited. For $T > 10^4$ K, the OPAL radiative opacities of Iglesias et al. (1992) were used, whereas for $T \leq 10^4$ K, the opacities of Kurucz (1991) were adopted. We have assumed a value $Z = 0.02$ for the solar metallicity and the heavy elements distribution as derived by Grevesse (1991). The solar helium abundance Y_{\odot} and the value of the mixing length parameter α have been derived by matching the luminosity and radius of a stellar model with $Z = 0.02$ and the solar age to their solar values. The values obtained are $\alpha = 2.25$ and $Y_{\odot} = 0.289$ — see Chieffi, Straniero & Salaris (1995) for a detailed discussion about the solar calibration with the new OPAL and Kurucz opacities.

Nuclear reaction rates have been taken from Fowler, Caughlan & Zimmerman (1975), and the subsequent modifications have been taken from Harris et al. (1983), Caughlan et al. (1985) and Caughlan & Fowler (1988). The reaction rate for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction is crucial for this study, since during the He burning phase, when central helium is depleted down to $Y = 0.10$, the burning mainly occurs through this reaction, and its rate determines the ^{12}C and ^{16}O profiles in the final white dwarf structure.

Several different teams have recently examined the cross section of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$

reaction (Ji et al. 1990, Zhao et al. 1993, Buchmann et al. 1993, Azuma et al. 1994, Mohr et al. 1995, Trautvetter 1996). A detailed analysis of the data obtained shows that these experiments are compatible with a total astrophysical S-factor at 300 keV (S_{300}) in the range $120 \text{ keV b} \lesssim S_{300} \lesssim 220 \text{ keV b}$ (Trautvetter 1996). There have also been several attempts to constrain the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate from astrophysical data. However, the fractions of ^{12}C and ^{16}O produced in a typical star depend both on the reaction rate and on the treatment of convection. Thus, since we cannot disentangle both effects, these constraints are only set on an *effective* cross section for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, given the lack of a reliable theory of convection.

Woosley, Timmes & Weaver (1993) studied the role of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate in producing the solar abundance set from stellar nucleosynthesis and concluded that the effective astrophysical S-factor for the energies involved during core helium burning that best reproduces the observed abundances should be $S_{300} = 170 \text{ keV b}$, in good agreement with the experimental data. This value for S_{300} corresponds to the value given by Caughlan & Fowler (1988) multiplied by ~ 1.3 . In their models the Ledoux criterion plus an amount of convective overshooting were adopted for determining the extension of the convective regions.

Thielemann, Nomoto & Hashimoto (1996) studied the collapse of gravitational supernovae and compared the predicted amount of ^{12}C and ^{16}O in their ejecta with the abundances observed in SN1987A and SN1993J. They found that the agreement was excellent when the reaction rate of Caughlan et al. (1985), which was computed assuming $S_{300} = 240 \text{ keV b}$, and the Schwarzschild criterion without overshooting were adopted.

Therefore, given our treatment of convection, we have adopted the rate of Caughlan et al. (1985) for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction. However, for a sake of comparison, two evolutionary sequences producing the most probable final white dwarf configurations —

that is white dwarf masses between 0.55 and 0.65 M_{\odot} — have also been computed using a lower cross section, namely the rate inferred by Woosley, Timmes & Weaver (1993) ($S_{300} = 170$ keV b, see above), hereinafter “low rate”.

3. The properties of the CO cores

With the input physics briefly described above, we have computed evolutionary sequences — neglecting mass loss — from the zero age main sequence to the thermally pulsing asymptotic giant branch (AGB) phase, of stellar models with masses in the range $1.0 \leq M/M_{\odot} \leq 7.0$. The tracks in the Hertzsprung-Russell diagram of these model sequences are shown in Figure 1, each one labeled with its corresponding mass.

EDITOR: PLACE FIGURE 1 HERE.

The evolutionary sequences were terminated at the end of the first thermal pulse. A real counterpart of these models is expected to lose its hydrogen-rich envelope during the thermally pulsing AGB phase, thanks to a rapid radiative wind, before becoming the central star of a planetary nebula, and finally evolve into a white dwarf. We have not explicitly followed these evolutionary phases since we are only interested in the chemical composition of the CO core and in the initial-final mass relation, in order to compute white dwarf cooling sequences (see below for details). As these phases can affect the initial-final mass relation (but not the ^{12}C and ^{16}O chemical profiles within the core), we have checked that our values for the mass internal to the He-H discontinuity (M_{WD}) as a function of the main sequence mass (M_{MS}) — see Table 1 — are compatible with the semi-empirical relation given by Weidemann (1987) and with that of Iben & Laughlin (1989). The maximum difference between our results and these two relations is of the order of 10%, well within the uncertainty associated to the semi-empirical determination. The time spent during the

pre-white dwarf phase (t) and the oxygen abundance at the center (X_O) at the end of the first thermal pulse are also displayed in Table 1. The maximum difference between our evolutionary times and those published by Iben & Laughlin (1989) — their equation 22 — is 1% in $\log t$.

EDITOR: PLACE TABLE 1 HERE.

Figure 2 displays the oxygen profiles for some of the CO cores obtained just at the end of the first thermal pulse ⁶. The inner part of the core, with a constant abundance of ^{16}O , is determined by the maximum extension of the central He-burning convective region while the peak in the oxygen abundance is produced when the He-burning shell crosses the semiconvective region partially enriched in ^{12}C and ^{16}O , and carbon is converted into oxygen through the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction. Beyond this region, the oxygen profile is built when the thick He-burning shell is moving towards the surface. Simultaneously, gravitational contraction increases its temperature and density, and since the ratio between the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ and 3α reaction rates is lower for larger temperatures — see e.g. Figure 1 in Mazzitelli & D’Antona (1987) — the oxygen mass fraction steadily decreases in the external part of the CO core. It is also interesting to notice that, in contrast with Mazzitelli and D’Antona (1987), all the models, including those of the highest mass, have their central regions dominated by oxygen (see Table 1). However, the amplitude of the peak in the oxygen abundance profile decreases as the mass of the CO core increases. Figure 2 also displays the ^{16}O profile (dotted line) of the white dwarf resulting from the evolution of a $3.2 M_\odot$ stellar model, computed adopting the low value of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate. The shape of the chemical profile is similar to that obtained by adopting the rate of Caughlan

⁶Detailed chemical abundance profiles for the CO cores described in this section are available upon request to the authors.

et al. (1985), but now, due to the less efficient conversion of ^{12}C into ^{16}O , the two elements have a more similar abundance in the inner part of the core ($X_{\text{C}}=0.40$, $X_{\text{O}}=0.57$).

EDITOR: PLACE FIGURE 2 HERE.

The ^{12}C and ^{16}O profiles at the end of the first thermal pulse have an off-centered peak in the oxygen profile, which is related to semiconvection (as explained above). Since we have chosen the rate of Caughlan et al. (1985) for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, we were forced to use the Schwarzschild criterion for convection (see the previous discussion in section 2) and, therefore, we did not find the chemical profiles to be Rayleigh-Taylor unstable during the early thermally-pulsing AGB phase. After the ejection of the envelope, when the nuclear reactions are negligible at the edge of the degenerate core, the Ledoux criterion can be used and, therefore, the chemical profiles are Rayleigh-Taylor unstable and, consequently, will be rehomogeneized by convection (Isern et al. 1996). Notice that, in any case, this rehomogeneization minimizes the effect of the separation occurring during the cooling process. Figure 3 shows the oxygen profile obtained for the $3.2 M_{\odot}$ model at the end of the first thermal pulse (dotted line), and the resulting profile after rehomogeneization (dotted-dashed line). The resulting profiles after Rayleigh-Taylor rehomogeneization are the initial profiles adopted in our cooling sequences.

EDITOR: PLACE FIGURE 3 HERE.

4. White dwarf cooling ages and luminosity functions

We have computed cooling sequences for the carbon-oxygen cores previously described according to the method developed by Díaz-Pinto et al. (1994) and subsequently modified

in García-Berro et al. (1996). This method assumes that the white dwarf has an isothermal core and that the luminosity is only a function of its mass and temperature. The adopted relationship between the core temperature and the luminosity is a fit to the results of Wood & Winget (1989) for a $0.6 M_{\odot}$ CO white dwarf, with a helium envelope of mass $10^{-4} M_{\text{WD}}$, conveniently scaled by mass, which is enough for our purposes. The cooling times and the characteristic cooling timescales are derived from the binding energy of the white dwarf and the aforementioned relationship. Our cooling sequences start at core temperatures of 5×10^7 K, which roughly correspond to luminosities $\sim 10^{-1} L_{\odot}$ — well below the knee in the Hertzsprung-Russell diagram. Neutrino cooling at high temperatures (i.e. high luminosities) has been included as in García-Berro et al. (1996). We have used the equation of state described in Segretain et al. (1994) which includes accurately all the relevant contributions to the thermodynamical quantities both in the liquid and in the solid phase. Phase separation during solidification has been included, using the phase diagram of the carbon-oxygen binary mixture of Segretain & Chabrier (1993), which is of the spindle form.

During the crystallization process of the white dwarf interior, the chemical composition of the solid and liquid phases are not equal. A solid, oxygen-rich core grows and the lighter carbon-rich fluid which is left ahead of the crystallization front is Rayleigh-Taylor unstable and is efficiently redistributed by convective motions in the outer liquid mantle (see Appendix A). The net effect is a migration of some oxygen towards the central regions which leads to a subsequent energy release (Mochkovitch 1983, Isern et al. 1996). The final profile for a $0.61 M_{\odot}$ white dwarf, when the whole interior has crystallized, is shown in Figure 3 as a solid line.

The cooling times as a function of the luminosity for the different models computed are shown in Table 2 and in Figure 4 ⁷. The onset of crystallization is clearly marked by

⁷Detailed cooling sequences are available upon request to the authors.

the change in the slope of the cooling curves. Obviously, massive white dwarfs crystallize at higher temperatures (luminosities) because they have larger central densities and their oxygen abundance does not vary significantly. As an example of the influence of phase separation in the cooling times, the time taken by a $0.61 M_{\odot}$ white dwarf to reach $\log(L/L_{\odot}) = -4.5$ is 9.9 Gyr (to be compared with 8.9 if phase separation is neglected). For comparison we have also computed the cooling sequence for the $0.6 M_{\odot}$ white dwarf obtained using the low $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate (see Figure 2 for the ^{16}O profile). The time necessary to reach $\log(L/L_{\odot}) \simeq -4.5$, is now 10.3 Gyr (to be compared with 9.2 Gyr if separation is neglected). Two aspects of these latter results deserve further discussion. First, either if phase separation is neglected or not, the cooling ages are larger for the low rate model sequence. This is due to their lower oxygen content, which leads to a larger heat capacity and, therefore, to a slower cooling rate. Second, the delay introduced by phase separation during crystallization down to $\log(L/L_{\odot}) = -4.5$ is practically the same for both cooling sequences (~ 1 Gyr). The reason for this is twofold: on one hand, in the model computed with the low rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, less oxygen is available for separation ($M_{\text{O}} = 0.3M_{\odot}$ instead of $M_{\text{O}} = 0.4M_{\odot}$, see Figure 2) but, on the other hand, the change in the oxygen abundance upon crystallization is larger, due to the spindle form of the phase diagram — see Figure 2 in Segretain et al. (1994).

In order to compare our new results for the cooling times with those of our previous works (Segretain et al. 1994, Hernanz et al. 1994), we should take into consideration the two basic improvements introduced since then. First of all, the profiles of chemical composition for the different possible progenitors of carbon-oxygen white dwarfs have been obtained using the best available effective rates for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction and updated input physics. For a typical $0.6 M_{\odot}$ white dwarf, the total oxygen mass is similar to that found by Mazzitelli & D’Antona (1986b), $M_{\text{O}} = 0.4 M_{\odot}$, but it is distributed in a different way. And second, we have improved the treatment of solidification, by taking into account the

actual profile instead of approximating it by a two-step function (see Segretain et al. 1994 and García-Berro et al. 1996). Our present treatment is more realistic and further reduces the effect of phase separation. We have recomputed the cooling sequence of a $0.6 M_{\odot}$ white dwarf using the exact initial chemical profile of Mazzitelli & D’Antona (1986b); the delay introduced by phase separation in the cooling time down to $\log(L/L_{\odot}) \simeq -4.5$ is now 0.9 Gyr instead of the previous 1.2 Gyr found in Segretain et al. (1994), which is similar to the value obtained with the chemical profiles derived in §3 (1 Gyr).

EDITOR: PLACE TABLE 2 HERE.

EDITOR: PLACE FIGURE 4 HERE.

Another important magnitude related to the cooling, which is directly involved in the calculation of the white dwarf luminosity function, is the characteristic cooling timescale, defined as $\tau_{\text{cool}} = dt_{\text{cool}}/dM_{\text{bol}}$. This quantity is shown in Figure 5 as a function of the luminosity, for the masses listed in Table 2. The onset of crystallization is clearly marked by a sudden increase of the τ_{cool} versus $\log(L/L_{\odot})$ relation, which corresponds to the change in the slope of the cooling curves. During the solidification of their interiors, white dwarfs must radiate away both the extra amount of energy due to the release of the latent heat of crystallization and the gravitational energy released by phase separation, thus slowing down the cooling process and consequently increasing the characteristic cooling timescales. The amplitude of the bump is smaller for massive white dwarfs because the release of energy at crystallization occurs at higher luminosities.

EDITOR: PLACE FIGURE 5 HERE.

Finally, in order to examine the influence of our cooling sequences on the estimation of the age of the solar neighborhood, we have computed white dwarf luminosity functions (see Figures 6 and 7), with the method explained in Hernanz et al. (1994), assuming a Salpeter-like initial mass function (Salpeter 1961) and a constant star formation rate per unit volume. The age of the disk that best fits the observational data of Liebert et al. (1988), when adopting blackbody corrections for the cool non-DA white dwarfs (see figure 6), is 9.3 Gyr — see Hernanz et al. (1994) for a discussion of the uncertainty of the age determination associated to the exact position of the cutoff ⁸. This age of the disk has to be compared with an age of 8.3 Gyr, obtained using the same set of inputs but neglecting phase separation. If the observational data set of Oswalt et al. (1996) is adopted (see Figure 7), our best fit corresponds to an age of 11.0 Gyr (10.0 Gyr if phase separation is neglected)

EDITOR: PLACE FIGURE 6 HERE.

EDITOR: PLACE FIGURE 7 HERE.

5. Conclusions

In this paper we have examined the influence of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate on the final structure of carbon-oxygen white dwarfs. The FRANEC evolutionary code, with updated input physics, has been used to derive accurate chemical profiles. The full

⁸For a general discussion, within the context of galactic evolution, of the uncertainties of age determinations using the white dwarf luminosity function, see Isern et al. (1995a, b) and Wood (1992).

range of initial masses producing carbon-oxygen white dwarfs as a final result, in the frame of single star evolution, has been analyzed. For the best choice of the combined effect of convection and the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate, carbon-oxygen profiles showing an enhancement of oxygen in the central regions for all core masses are obtained, whereas for a lower $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate, this effect is smaller. Mass fractions of ^{16}O as high as 0.83 are obtained for a $0.55 M_{\odot}$ white dwarf, descending from a $2.5 M_{\odot}$ star, whereas $X_{\text{O}} = 0.66$ corresponds to a $1.0 M_{\odot}$ white dwarf, descending from a $7 M_{\odot}$ star.

The resulting carbon-oxygen profiles have been used for computing white dwarf cooling sequences, including the effect of phase separation during solidification. This phenomenon leads to a non negligible increase of the cooling ages, which translates into an increase of the age of the disk of the same order. Our best estimate of the age of the disk is 9.3 Gyr, when the data set of Liebert et al. (1988) is used, and 11.0 Gyr when the data set of Oswalt et al. (1996) is adopted, in contrast with 8.3 and 10.0 Gyr, obtained, respectively, when phase separation is neglected. These values indicate that the effects associated with crystallization should not be neglected when using white dwarfs as a tool to determine the age of the disk.

This work has been supported by DGICYT grants PB94-0111 and PB94-0827-C02-02, by the CIRIT grant GRQ94-8001, by the AIHF 335-B, by the AIHI 94-082-A and by the C⁴ consortium. One of us (M.S.) thanks the E.C. for the “Human Capital and Mobility” fellowship ERBCHGECT920009.

Appendix A: Change of the chemical profile during the solidification process

The distribution of carbon and oxygen in the outer liquid mantle of a crystallizing white dwarf is Rayleigh-Taylor unstable because a lighter carbon-rich fluid is released at the crystallization boundary. Convective mixing will redistribute the abundances and lead to flat profiles in a region whose size depends on the initial composition profile and on the degree of enrichment produced during the solidification process.

Consider, therefore, a partially solidified white dwarf of total mass M_{WD} containing a total amount of oxygen M_{O} . Its structure can be divided into three parts: a solid core of mass M_{S} and oxygen mass fraction $X_{\text{S}}(m)$, a liquid mantle of mass ΔM homogenized by convection, with oxygen abundance X , and an outer, unperturbed region, with the initial oxygen profile $X_{\text{O}}(m)$. Therefore, the total mass of oxygen can be written as

$$M_{\text{O}} = \int_0^{M_{\text{S}}} X_{\text{S}} dm + X(M_{\text{L}} - M_{\text{S}}) + \int_{M_{\text{L}}}^{M_{\text{WD}}} X_{\text{O}} dm \quad (\text{A.1})$$

where $M_{\text{L}} = M_{\text{S}} + \Delta M$

After deriving this expression with respect to the solid mass and introducing the two conditions $X_{\text{S}}(M_{\text{S}}) = (1 + \alpha)X$ and $X = X_{\text{O}}(M_{\text{S}})$, where α , which depends on X , is the degree of enrichment produced during crystallization, we obtain:

$$\alpha X + \frac{dX}{dM_{\text{S}}}(M_{\text{L}} - M_{\text{S}}) = 0 \quad (\text{A.2})$$

introducing $q = M_{\text{S}}/M_{\text{WD}}$ and $q_{\text{L}} = M_{\text{L}}/M_{\text{WD}}$,

$$\frac{dX}{dq}[q_{\text{L}}(X) - q] + \alpha(X)X = 0 \quad (\text{A.3})$$

Notice the singularity at $q = 0$ since $q_{\text{L}}(X) = 0$ and also notice that if the initial profile

is flat, $q_L(X) = 1$. Integrating this equation provides the final oxygen profile after crystallization.

REFERENCES

- Azuma, R.E., Buchmann, L., Barker, F.C., Barnes, C.A. 1994, *Phys. Rev. C*, 50, 1194
- Buchmann, L., Azuma, R.E., Barnes, C.A., D’Auria, J.M., Dombsky, M., Giesen, M., Jackson, K.P., King, J.D., Korteling, R.G., McNeely, P., Powel, J., Roy, G., Vicent, J., Wang, T.R., Wrean, P.R. 1993, *Phys. Rev. C*, 726
- Castellani, V., Chieffi A., Pulone, L., Tornambé, A. 1985, *ApJ*, 296, 204
- Caughlan, G.R., Fowler, W.A., Harris, M.J., Zimmermann, B.A. 1985, *Atomic Data & Nuclear Data Tables*, 32, 197
- Caughlan, G.R., Fowler, W.A. 1988, *Atomic Data & Nuclear Data Tables*, 36, 411
- Chieffi A., Straniero, O. 1989, *ApJS*, 71, 47
- Chieffi A., Straniero, O., Salaris, M. 1995, *ApJ*, 445, L39
- Díaz-Pinto, A., García-Berro, E., Hernanz, M., Isern, J., Mochkovitch, R. 1994, *A&A*, 282, 86
- Fowler, W.A., Caughlan, G.R., Zimmermann, B.A. 1975, *ARA&A*, 13, 69
- García-Berro, E., Hernanz, M., Isern, J., Chabrier, G., Segretain, L., Mochkovitch, R. 1996, *A&AS*, 117, 12
- García-Berro, E., Hernanz, M., Mochkovitch, R., Isern, J. 1988, *A&A*, 193, 141
- Grevesse, N. 1991, *A&A*, 242, 488
- Harris, M.J., Fowler, W.A., Caughlan, G.R., Zimmermann, B.A. 1983, *ARA&A*, 21, 165
- Hernanz, M., García-Berro, E., Isern, J., Mochkovitch, R., Segretain, L., Chabrier, G. 1994, *ApJ*, 434, 652

- Iben, I., Laughlin, G. 1989, ApJ, 341, 312
- Iglesias, C. A., Rogers, F. J., Wilson, B. G. 1992, ApJ, 397, 717
- Isern, J., García-Berro, E., Hernanz, M., Mochkovitch, R., and Burkert, A. 1995a, in *White Dwarfs*, ed. D. Koester and K. Werner (Springer), 19.
- Isern, J., García-Berro, E., Hernanz, M., Mochkovitch, R., and Burkert, A. 1995b, in *The Formation of the Milky Way*, ed. E.J. Alfaro and A.J. Delgado (CUP), 19.
- Isern, J., Mochkovitch, R., García-Berro, E., and Hernanz, M., ApJ, submitted
- Ji, X., Filippone, B.W., Koonin, S.E. 1990, Phys. Rev. C, 41, 1736
- Koester, D., Chanmugam, G. 1990, Rep. Prog. Phys. 54, 837
- Kurucz, R. L. 1991, in *Stellar Atmospheres: Beyond Classical Models*, NATO ASI Series C, Vol. 341
- Liebert, J., Dahn, C.C., Monet, D.G. 1988, ApJ, 332, 891
- Mazzitelli I., D’Antona F. 1986a, ApJ, 308, 706
- Mazzitelli I., D’Antona F. 1986b, ApJ, 311, 762
- Mazzitelli I., D’Antona F. 1987, in *IAU Colloquium # 95: The Second Conference on Faint Blue Stars*, ed. A.G.D. Philip, S. Hayes, and J. Liebert (Schenectady: Davis Press), 351
- Mochkovitch, R. 1983, A&A122, 212
- Mohr, P., Kölle, V., Wilmes, S., Atzrott, U., Hoyler, F., Engelman, C., Staudt, G., Oberhummer, H. 1995, in *Nuclei in the Cosmos*, Vol. 3, ed. R. Busso, C. Raiteri, and R. Gallino (New York: AIP), in press

- Oswalt, T.D., Smith, J.A., Wood, M.A., Hintzen, P. 1996, *Nature*, 382, 692.
- Segretain, L., Chabrier, G. 1993, *A&A*, 271, L13
- Segretain, L., Chabrier, G., Hernanz, M., García-Berro, E., Isern, J., Mochkovitch, R. 1994, *ApJ*, 434, 641
- Salpeter, E.E. 1961, *ApJ*, 134, 669
- Thielemann, F.-K., Nomoto, K., Hashimoto, M. 1996, *ApJ*, 460, 408
- Trautvetter, K. 1996, preprint.
- Wood M.A. 1992, *ApJ*, 386, 539
- Wood M.A., Winget D.E. 1989, in *IAU Colloq. 114, White Dwarfs*, ed. G. Wegner (Berlin: Springer), 282
- Weidemann V. 1987, *A&A*188, 74
- Woosley, S.E., Timmes, F.X., Weaver T.A. 1993, in *Nuclei in the Cosmos*, Vol. 2, ed. F. Käppeler and K. Wisshak (IOP Publishing Ltd), 531
- Zhao, Z., France, R.H. III, Lai, K.S., Rugari, L., Gai, M. 1993, *Phys. Rev. C*, 70, 2066

Fig. 1.— Evolutionary tracks in the Hertzsprung-Russell diagram for the model sequences quoted in the text. Each track is labeled with its corresponding mass.

Fig. 2.— Oxygen profiles for selected white dwarf models with masses of 0.61, 0.68 and 0.87 M_{\odot} and our choice of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate (solid line). The dotted line displays the oxygen profile for the 0.60 M_{\odot} model computed by adopting the low rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction.

Fig. 3.— Oxygen profile of a 0.61 M_{\odot} white dwarf at the beginning of the thermally-pulsing AGB phase (dotted line), the same after rehomogenization by Rayleigh-Taylor instabilities during the liquid phase (dotted-dashed line) and after total freezing (solid line).

Fig. 4.— Cooling curves (time is in Gyr) for the white dwarf models described in the text and in Table 1 (except for the “low rate” case).

Fig. 5.— Characteristic cooling times (in yr) for the same models shown in Figure 4.

Fig. 6.— Luminosity function obtained assuming a constant star formation rate per unit volume and an age of the disk of 9.3 Gyr. The observational data are from Liebert, Dahn and Monet (1988).

Fig. 7.— Luminosity function obtained assuming a constant star formation rate per unit volume and an age of the disk of 11.0 Gyr. The observational data are from Oswalt, Smith, Wood and Hintzen (1996).

Table 1. Characteristics of the white dwarfs obtained adopting the rate of Caughlan et al. (1985) for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction

$M_{\text{MS}}(M_{\odot})$	$\log t \text{ (yr)}$	$M_{\text{WD}}(M_{\odot})$	X_{O}
1.0	10.0	0.54	0.79
1.5	9.36	0.54	0.79
2.0	9.02	0.54	0.79
2.5	8.88	0.55	0.83
3.2	8.56	0.61	0.74
3.6	8.41	0.68	0.72
4.0	8.27	0.77	0.71
5.0	8.01	0.87	0.68
7.0	7.66	1.00	0.66

Table 2. Cooling times, in Gyr, for white dwarfs of different masses.

$-\log(L/L_{\odot})$	t_{cool}							
	$0.54 M_{\odot}$	$0.55 M_{\odot}$	$0.61 M_{\odot}$	$0.60 M_{\odot}^{\text{a}}$	$0.68 M_{\odot}$	$0.77 M_{\odot}$	$0.87 M_{\odot}$	$1.00 M_{\odot}$
2.00	0.10	0.10	0.12	0.12	0.16	0.21	0.28	0.42
2.20	0.17	0.18	0.22	0.22	0.27	0.33	0.42	0.60
2.40	0.28	0.29	0.34	0.34	0.40	0.48	0.60	0.81
2.60	0.42	0.43	0.48	0.49	0.56	0.67	0.80	1.06
2.80	0.58	0.60	0.67	0.68	0.76	0.89	1.05	1.39
3.00	0.79	0.81	0.89	0.91	1.02	1.17	1.37	1.98
3.20	1.05	1.08	1.18	1.21	1.34	1.54	1.89	2.81
3.40	1.39	1.43	1.57	1.60	1.77	2.11	2.79	3.83
3.60	1.85	1.90	2.09	2.14	2.58	3.16	4.02	5.03
3.80	2.66	2.73	3.19	3.02	3.91	4.68	5.57	6.36
4.00	4.09	4.22	4.86	4.91	5.72	6.58	7.30	7.67
4.20	5.94	6.20	6.87	7.06	7.80	8.50	8.86	8.69
4.40	7.98	8.30	9.01	9.22	9.60	10.00	9.99	9.40
4.50	8.88	9.22	9.85	10.26	10.33	10.59	10.45	9.69
4.60	9.69	10.02	10.60	11.25	10.97	11.12	10.86	9.95
4.70	10.43	10.75	11.27	12.21	11.56	11.61	11.25	10.20

^alow rate













